ISSN 1392 - 1207. MECHANIKA. 2011. 17(5): 545-550

Mathematical model of the operation of a weight batcher for dry products

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1. Introduction

Intensive development of packing technique encourages development of batchers [1-3]. It promotes to necessity increase a speed of dosing process and to necessity increase an accuracy of dosing [4-6]. Dry materials are widely popular materials which have dosed and packed. Sugar, salt, cereals are the dry materials of food products. They dosage and pack fertilizer, fastener, various component and other similar goods they materials have assigned as dry materials. Volumetric batchers are widely using for dosage the dry products [7]. Volumetric batchers usually have high speed of action. However their dosing precision is more dependent of the product's properties. In addition to volumetric batchers there are use weight batchers too [6-8]. The dosing precision of weight batchers is small dependent of product's properties. Using the weight batcher, you can choose a desirable proportion between speed of dosing and accuracy of dosing. However, the work cycle of weight batcher is more complex. In the paper we will theoretically investigate the work of weight batchers. The mathematical model describes the work of weight batchers for dry products. The sensitive element of weight batchers is regarded as elastic system with one degree of freedom. The body of variable mass is operating to the elastic system [8-11].

2. Structural scheme and physical model for batcher of weight

The weight batcher can achieve the high accuracy of dosing. However, the structure of weight batcher is complex. All components of the batcher affect the dosing accuracy [2, 3, 5, 6, 8]. The weight batcher investigation can begin from their typical structural scheme. It is typical structural scheme the batcher of weight (Fig. 1), where *1* - elastic element; *2* - shovel; *3* - sensor; *4* - control unit; *5* - unloading mechanism; *6* - feeder. The batcher of weight works as follows. Feeder *6* filed the product to the shovel *2*, the weight of shovel *2* increases, so elastic element *1* distort, it affects the sensor *3*, signal from the sensor *3* is transferred to the control unit *4*, which disables the feeder *6*, when the quantity of the product in shovel *2* is such as the asked the value. When the feeder 6 is stop, the control unit *4* starts the unloading mechanism *5* and the dose is loss. After a pause the batcher of weight is ready to work again. The batcher of weight combines several processes. These are vibratory transport, weight measurement, preparation, assessment and control technology. All these processes need to be investigated individually. It will also evaluate their interaction. Vibratory transport and vibratory processes are widely used [12-14]. The elastic systems are being payed a lot of attention [15, 16].

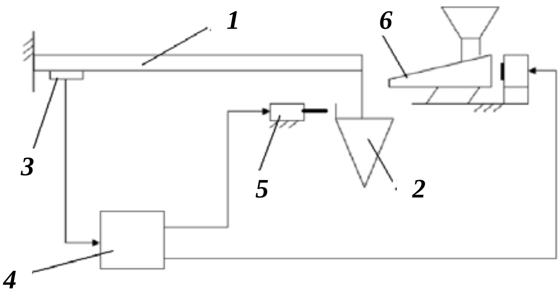

Fig. 1 Typical structural scheme the batcher of weight

The physical model of dosing process regarded as elastic system with one degree of freedom (Fig. 2), where *1* - elastic element with the elastic coefficient *k*; *2* - rigid body with the mass m_1; *3* - damping element with the coefficient of damping *c*; *4* - filed product with the mass Δm, *5* - discard product with the mass $\Delta_1 m$.

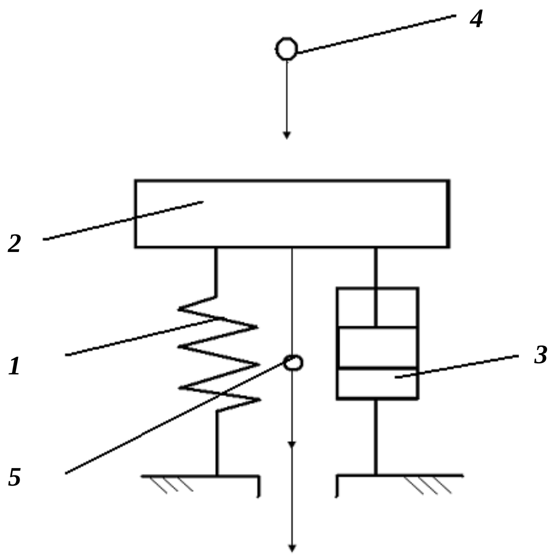

Fig. 2 Physical model of dosing process

The working cycle of weight batcher consists of five stages. The first step is the fill with the increased productivity. The second is the fill with the low productivity. The third - the fill is suspended (pause before discharge). The fourth step is discharge of the product. The fifth - the discharge is suspended (pause after discharge). Each of these phases is associated with the characteristics of the product, with the accuracy of dosing and with the efficiency of dosing as well as with other parameters of this system. Such as dosing rate, the systems sensitivity and working range, sensor type, the influence of technological regimes, and others.

When the elastic element is the console with mounted mass M, it reduced mass of shovel [15]

$$m_1 = M + 33m'/40 \tag{1}$$

where m_1 is reduced mass of shovel, M is mass of shovel, m' is mass of elastic element.

3. Mathematical model for describing the operation of weight batcher for dry products

This article deals with a movement of sensitive element at all five stages. As already mentioned, in this case we deal with the system with one degree of freedom, on this system to operate the body of variable mass. Mass of the dosing product in the shovel increases by linear law $m = \kappa_1 t$. Where t is time, κ_1 is constant coefficient (efficiency of feeder). The duration of operation the feeder depends on the dose size to be set up at this stage. Feeder working range in the first stage $t \in [0 : t_1]$, where $t_1 = \mu_1/\kappa_1$ is duration of the feeder work required to form a dose μ_1.

The formula of dynamics for body of variable mass is known [9]. This formula lets find the force which acts on the shovel in this case.

$$N = \kappa_1 u_1 - \kappa_1 v - \kappa_1 g t - \kappa_1 t\, dv/dt \tag{2}$$

where u_1 is the absolute velocity of joining particle, v is the absolute velocity of shovel, dv/dt is the absolute acceleration of the shovel, g is the free fall acceleration. With this assess it is possible to write a differential equation, which describes movement of the shovel [9, 17].

$$m_1\ddot{x} = -kx - c\dot{x} - m_1 g + N \tag{3}$$

Eqs. (1) and (2) gives the equation

$$(m_1 + \kappa_1 t)\ddot{x} + (c + \kappa_1)\dot{x} + kx = m_1 g + \kappa_1 u_1 + \kappa_1 g t \tag{4}$$

With the change of variables

$$\left.\begin{aligned} x &= y + \kappa_1 u_1/k - (c+\kappa_1)\kappa_1 g/k^2 + (t + m_1/\kappa_1)\kappa_1 g/k \\ t &= \xi_1 - m_1/\kappa_1 \end{aligned}\right\} \tag{5}$$

Equation (4) gives the equation

$$\xi_1 \ddot{y} + (1 + c/\kappa_1)\dot{y} + k/\kappa_1\, y = 0 \tag{6}$$

where solution is expressed in the first kind J_n and second kind Y_n of Bessel functions [18-20].

$$y = \xi_1^{-n/2}\left[C_1 J_n\left(2\sqrt{\xi k/\kappa_1}\right) + C_2 Y_n\left(2\sqrt{\xi k/\kappa_1}\right)\right] \tag{7}$$

where C_1 and C_2 are constants. Returning to the previous variables, we obtain

$$\begin{aligned} x &= a_1 + gt/b_1 + (t+\tau_1)^{-n/2} \times \\ &\times \left\{C_1 J_n\left[2\sqrt{b_1(t+\tau_1)}\right] + C_2 Y_n\left[2\sqrt{b_1(t+\tau_1)}\right]\right\} \end{aligned} \tag{8}$$

where

$$\left.\begin{aligned} a_1 &= (\kappa_1 u_1 + m_1 g)1/k - (\kappa_1 + c)\kappa_1 g/k^2 \\ b_1 &= k/\kappa_1,\ \tau_1 = m_1/\kappa_1,\ n = c/\kappa_1,\ u_1 = \sqrt{2hg} \end{aligned}\right\} \tag{9}$$

Derivative of Eq. (8)

$$\begin{aligned} \dot{x} &= \frac{g}{b_1} - \sqrt{b_1}\,(t+\tau_1)^{-\frac{n+1}{2}} \times \\ &\times \left\{C_1 J_{n+1}\left[2\sqrt{b_1(t+\tau_1)}\right] + C_2 Y_{n+1}\left[2\sqrt{b_1(t+\tau_1)}\right]\right\} \end{aligned} \tag{10}$$

When the initial conditions of movement $t = 0$, $x = e = m_1 g/k$, $\dot{x} = 0$ we get that

$$\begin{cases} C_1 J_n\left(2\sqrt{b_1\tau_1}\right) + C_2 Y_n\left(2\sqrt{b_1\tau_1}\right) = (e - a_1)\tau_1^{\frac{n}{2}} \\ C_1 J_{n+1}\left(2\sqrt{b_1\tau_1}\right) + C_2 Y_{n+1}\left(2\sqrt{b_1\tau_1}\right) = \left(g/b_1\sqrt{b_1}\right)\tau_1^{\frac{n+1}{2}} \end{cases} \tag{11}$$

It follows

$$\Delta = \begin{vmatrix} J_n\left(2\sqrt{b_1\tau_1}\right) & Y_n\left(2\sqrt{b_1\tau_1}\right) \\ J_{n+1}\left(2\sqrt{b_1\tau_1}\right) & Y_{n+1}\left(2\sqrt{b_1\tau_1}\right) \end{vmatrix} \tag{12}$$

$$\Delta_1 = \begin{vmatrix} (e - a_1)\tau_1^{\frac{n}{2}} & Y_n\left(2\sqrt{b_1\tau_1}\right) \\ \tau_1^{\frac{n+1}{2}} g/b_1\sqrt{b_1} & Y_{n+1}\left(2\sqrt{b_1\tau_1}\right) \end{vmatrix} \tag{13}$$

$$\Delta_2 = \begin{vmatrix} J_n\left(2\sqrt{b_1\tau_1}\right) & (e - a_1)\tau_1^{\frac{n}{2}} \\ J_{n+1}\left(2\sqrt{b_1\tau_1}\right) & \tau_1^{\frac{n+1}{2}} g/b_1\sqrt{b_1} \end{vmatrix} \tag{14}$$

and

$$\begin{aligned} \Delta &= J_n\left(2\sqrt{b_1\tau_1}\right) Y_{n+1}\left(2\sqrt{b_1\tau_1}\right) - \\ &- J_{n+1}\left(2\sqrt{b_1\tau_1}\right) Y_n\left(2\sqrt{b_1\tau_1}\right) \end{aligned} \tag{15}$$

According to [18-20], we obtain

$$\Delta = -1/\pi\sqrt{b_1\tau_1} \tag{16}$$

$$\Delta_1 = \left(e - a_1\right)\tau_1^{\frac{n}{2}} Y_{n+1}\left(2\sqrt{b_1\tau_1}\right) - \left(g/b_1\sqrt{b_1}\right)\tau_1^{\frac{n+1}{2}} Y_n\left(2\sqrt{b_1\tau_1}\right) \quad (17)$$

$$\Delta_2 = \frac{g}{b_1\sqrt{b_1}}\tau_1^{\frac{n+1}{2}} J_n\left(2\sqrt{b_1\tau_1}\right) - \left(e - a_1\right)\tau_1^{\frac{n}{2}} J_{n+1}\left(2\sqrt{b_1\tau_1}\right) \quad (18)$$

Since $C_1 = \Delta_1/\Delta$; $C_2 = \Delta_2/\Delta$, evaluated the equations (16)-(18) gives

$$C_1 = \frac{\pi g}{b_1}\tau_1^{1+\frac{n}{1}} Y_n\left(2\sqrt{b_1\tau_1}\right) - \\ -\pi\sqrt{b_1}\left(e - a_1\right)\tau_1^{\frac{n+1}{2}} Y_{n+1}\left(2\sqrt{b_1\tau_1}\right) \quad (19)$$

$$C_2 = -\frac{\pi g}{b_1}\tau_1^{1+\frac{n}{2}} J_n\left(2\sqrt{b_1\tau_1}\right) + \\ +\pi\sqrt{b_1}\left(e - a_1\right)\tau_1^{\frac{n+1}{2}} J_{n+1}\left(2\sqrt{b_1\tau_1}\right) \quad (20)$$

The second step is the fill with the low productivity. Mass of the dosing product in the shovel increases by linear law $m = \kappa_2 t$. Where t is time, κ_2 is constant coefficient (efficiency of feeder). The duration of work of the feeder depends on the dose size to be set up at this stage. The feeder working range in the second stage $t \in [0 : t_2]$, where $t_2 = \mu_2/\kappa_2$ - duration of the feeder work required to form a dose μ_2. It is necessary assess the change mass of the shovel. At this stage reduced mass of the shovel [15]. $m_2 = M + \kappa_1 t_1 + \frac{33}{140} m'$ where m_2 is reduced mass of the shovel in the second step. In the second step the differential equation this describes the movement of the shovel.

$$\left(m_2 + \kappa_2 t\right)\ddot{x} + \left(c + \kappa_2\right)\dot{x} + kx = m_2 g + \kappa_2 u_2 + \kappa_2 gt \quad (21)$$

where u_2 is the absolute velocity of joining particle in the second step. The solution [18-20] of Eq. (21) is

$$x = a_2 + gt/b_2 + \left(t + \tau_2\right)^{-n/2} \times \\ \times\left\{C_1' J_n\left[2\sqrt{b_2\left(t + \tau_2\right)}\right] + C_2' Y_n\left[2\sqrt{b_2\left(t + \tau_2\right)}\right]\right\} \quad (22)$$

where

$$\left.\begin{aligned} a_2 &= \left(\kappa_2 u_2 + m_2 g\right)1/k - \left(\kappa_2 + c\right)\kappa_2 g/k^2 \\ b_2 &= k/\kappa_2,\ \tau_2 = m_2/\kappa_2,\ n = c/\kappa_2,\ u_2 = \sqrt{2h_2 g} \end{aligned}\right\} \quad (23)$$

where J_n is the first kind and Y_n is second kind of Bessel functions, C_1' and C_2' are constants. Derivative of Eq. (22) is

$$\dot{x} = g/b_2 - \sqrt{b_2}\left(t + \tau_2\right)^{-\frac{n+1}{2}} \times \\ \times\left\{C_1' J_{n+1}\left[2\sqrt{b_2\left(t + \tau_2\right)}\right] + C_2' Y_{n+1}\left[2\sqrt{b_2\left(t + \tau_2\right)}\right]\right\} \quad (24)$$

When $t = 0$, from Eqs. (22), (24) we get that

$$x = a_2 + \tau_2^{-n/2}\left\{C_1' J_n\left[2\sqrt{b_2\tau_2}\right] + C_2' Y_n\left[2\sqrt{b_2\tau_2}\right]\right\} \quad (25)$$

$$\dot{x} = g/b_2 - \sqrt{b_2}\left(\tau_2\right)^{-\frac{n+1}{2}} \times \\ \times\left\{C_1' J_{n+1}\left[2\sqrt{b_2\tau_2}\right] + C_2' Y_{n+1}\left[2\sqrt{b_2\tau_2}\right]\right\} \quad (26)$$

At the same time movement of the shovel is described by the equations (7), (10) at the first phase when $t = t_1$,

$$x_1 = a_1 + gt_1/b_1 + \left(t_1 + \tau_1\right)^{-n/2} \times \\ \times\left\{C_1 J_n\left[2\sqrt{b_1\left(t_1 + \tau_1\right)}\right] + C_2 Y_n\left[2\sqrt{b_1\left(t_1 + \tau_1\right)}\right]\right\} \quad (27)$$

$$\dot{x}_1 = g/b_1 - \sqrt{b_1}\left(t_1 + \tau_1\right)^{-\frac{n+1}{2}} \times \\ \times\left\{C_1 J_{n+1}\left[2\sqrt{b_1\left(t_1 + \tau_1\right)}\right] + C_2 Y_{n+1}\left[2\sqrt{b_1\left(t_1 + \tau_1\right)}\right]\right\} \quad (28)$$

From Eqs. (25), (27) and (26), (28) we obtain

$$\begin{cases} C_1' J_n\left(2\sqrt{b_2\tau_2}\right) + C_2' Y_n\left(2\sqrt{b_2\tau_2}\right) = \left(x_1 - a_2\right)\tau_2^{\frac{n}{2}} \\ C_1' J_{n+1}\left(2\sqrt{b_2\tau_2}\right) + C_2' Y_{n+1}\left(2\sqrt{b_2\tau_2}\right) = \frac{1}{\sqrt{b_2}}\tau_2^{\frac{n+1}{2}}\left(\frac{g}{b_2} - \dot{x}_1\right) \end{cases} \quad (29)$$

With the change of variables

$$\left.\begin{aligned} R_2 &= \left(x_1 - a_2\right)\tau_2^{\frac{n}{2}} \\ S_2 &= \tau_2^{\frac{n+1}{2}}\left(g/b_2 - \dot{x}_1\right)1/\sqrt{b_2} \end{aligned}\right\} \quad (30)$$

gives

$$\Delta = J_n\left(2\sqrt{b2\tau_2}\right)Y_{n+1}\left(2\sqrt{b_2\tau_2}\right) - \\ -J_{n+1}\left(2\sqrt{b_2\tau_2}\right)Y_n\left(2\sqrt{b_2\tau_2}\right) \quad (31)$$

According to [18-20], we obtain

$$\Delta = -1/\pi\sqrt{b_2\tau_2} \quad (32)$$

$$\Delta_1 = R_2 Y_{n+1}\left(2\sqrt{b_2\tau_2}\right) - S_2 Y_n\left(2\sqrt{b_2\tau_2}\right) \quad (33)$$

$$\Delta_2 = S_2 J_n\left(2\sqrt{b_2\tau_2}\right) - R_2 J_{n+1}\left(2\sqrt{b_2\tau_2}\right) \quad (34)$$

Since $C_1' = \Delta_1/\Delta$; $C_2' = \Delta_2/\Delta$, evaluating of the Eqs. (32)-(34) we get

$$C_1' = \pi\tau_2^{1+\frac{n}{2}}\left(g/b_2 - \dot{x}_1\right)Y_n\left(2\sqrt{b_2\tau_2}\right) - \\ -\pi\sqrt{b_2}\left(x_1 - a_2\right)\tau_2^{\frac{n+1}{2}} Y_{n+1}\left(2\sqrt{b_2\tau_2}\right) \quad (35)$$

$$C_2' = -\pi\tau_2^{1+\frac{n}{2}}\left(g/b_2 - \dot{x}_1\right)J_n\left(2\sqrt{b_2\tau_2}\right) + \\ + \pi\sqrt{b_2}\left(x_1 - a_2\right)\tau_2^{\frac{n+1}{2}} J_{n+1}\left(2\sqrt{b_2\tau_2}\right) \quad (36)$$

The third stage starts at the moment when the fill is suspended (pause before discharge). At this moment the shovel mass $\mu = \kappa_1 t_1 + \kappa_2 t_2$.

Differential duration, which describes movement of the shovel at stage 3 [15] is

$$(m_1 + \mu)\ddot{x} + c\dot{x} + kx = (m_1 + \mu)g \quad (37)$$

In step 3 $t \in [0 : t_3]$, where t_3 the pause time.

At initial shovel movement conditions $t = 0$, $x = x_2$, $\dot{x} = \dot{x}_2$. According to [15], solution of Eq. (37), at $\omega_0 > \vartheta$ shall be

$$x = g/k\left(m_1 + \mu\right) + Me^{-\vartheta t} sin\left(\omega t + N\right) \quad (38)$$

Derivative of Eq. (38)

$$\dot{x} = -M\vartheta e^{-\vartheta t} sin\left(\omega t + N\right) + M\omega e^{-\vartheta t} cos\left(\omega t + N\right) \quad (39)$$

where

$$\left.\begin{aligned} & 2\vartheta = c/(m_1 + \mu), \omega_0^2 = k/(m_1 + \mu), \omega^2 = \omega_0^2 + \vartheta^2 \\ & M^2 = [x_2 - (m_1 + \mu)g/k]^2 + \\ & + \{\dot{x}_2/\omega + [x_2 - (m_1 + \mu)g/k]\vartheta/\omega\}^2 \\ & tgN = \frac{x_2 - (m_1 + \mu)g/k}{\dot{x}_2/\omega + [x_2 - (m_1 + \mu)g/k]\vartheta/\omega} \end{aligned}\right\} \quad (40)$$

and accordingly at $\omega_0 < \vartheta$ Solution of same Eq. (37) shall be

$$x = \left(m_1 + \mu\right)g/k + e^{-\vartheta t}\left(M'e^{\omega t} + N'e^{-\omega t}\right) \quad (41)$$

Derivative of Eq. (41)

$$\dot{x} = M'(\omega - \vartheta)e^{(\omega-\vartheta)t} - N'(\omega + \vartheta)e^{-(\omega+-\vartheta)t} \quad (42)$$

In this case,

$$\left.\begin{aligned} & \omega^2 = \vartheta^2 - \omega_0^2, M' = \frac{\dot{x}_2}{2\omega} + \frac{\omega + \vartheta}{2\omega}\left[x_2 - \frac{g}{k}\left(m_1 + \mu\right)\right] \\ & N' = -\frac{\dot{x}_2}{2\omega} + \frac{\omega - \vartheta}{2\omega}\left[x_2 - \frac{g}{k}\left(m_1 + \mu\right)\right] \end{aligned}\right\} \quad (43)$$

Dimensions x_2 and $\dot{x}_2$ are displacement and speed of the shovel at which ends of the filing of the second stage. These values are derived from Eqs. (27), (28) when $t = t_2$.

The fourth step is discharge of the product. Mass of the product being discharged from the shovel decreases at linear law $m = -\kappa_3 t$. Here t is time, κ_3 is constant coefficient representing the discharge capacity. The duration of discharge depends on the dose size, i.e. $\mu = \mu_1 + \mu_2$. Working range at stage 4 is $t \in [0 : t_4]$, where $t_4 = \mu/\kappa_3$ is the time needed to fully discharge the dose μ.

In step 4 a differential equation, which describes movement of the shovel is

$$\left(m_1 + \mu - \kappa_3 t\right)\ddot{x} + c\dot{x} + kx = \left(m_1 + \mu\right)g + \kappa_3 gt \quad (44)$$

After replacement of variables

$$\left.\begin{aligned} & x = y + gc\kappa_3/k^2 - g/k\left(m_1 + \mu\right) - t\kappa_3 g/k \\ & t = -\xi_3 - \left(m_1 + \mu\right)/\kappa_3 \end{aligned}\right\} \quad (45)$$

in the Eq. (44) we get

$$\xi_3\ddot{y} - \dot{y}c/\kappa_3 + yk/\kappa_3 = 0 \quad (46)$$

According to [18] the soliution of Eq. (46) is

$$y = \xi_3^{\frac{1+i}{2}}\left[C_1'' J_{1+i}\left(2\sqrt{\xi_3 k/\kappa_3}\right) + C_2'' Y_{1+i}\left(2\sqrt{\xi_3 k/\kappa_3}\right)\right] \quad (47)$$

where J_{1+i} is the first kind and Y_{1+i} is second kind of Bessel functions, C_1'' and C_2'' are constant, $i = \frac{c}{\kappa_3}$.

Returning to the previous variables and taking into account that

$$\left.\begin{aligned} & a_3 = gc\kappa_3/k^2 + \left(m_1 + \mu\right)g/k \\ & b_3 = k/\kappa_3, \quad \sigma = \left(m_1 + \mu\right)/\kappa_3 \end{aligned}\right\} \quad (48)$$

we obtain

$$x = a_3 - gt/b_3 + \left(\sigma - t\right)^{\frac{1+i}{2}} \times \\ \times\left\{C_1'' J_{1+i}\left[2\sqrt{b_3\left(\sigma - t\right)}\right] + C_2'' Y_{1+i}\left[2\sqrt{b_3\left(\sigma - t\right)}\right]\right\} \quad (49)$$

Derivative of Eq. (49)

$$\dot{x} = -g/b_3 - \left(1 + i\right)\left(\sigma - t\right)^{\frac{i+1}{2}} \times \\ \times Z_{1+i}\left[2\sqrt{b_3\left(\sigma - t\right)}\right] - \left(\sigma - t\right)^{\frac{i}{2}}\sqrt{b_3} Z_{2+i}\left[2\sqrt{b_3\left(\sigma - t\right)}\right] \quad (50)$$

where

$$Z_n\left(x\right) = C_1'' J_n\left(x\right) + C_2'' Y_n\left(x\right) \quad (51)$$

When $t = 0$

$$x = a_3 + \sigma^{\frac{1+i}{2}}\left\{C_1'' J_{1+i}\left[2\sqrt{b_3\sigma}\right] + C_2'' Y_{1+i}\left[2\sqrt{b_3\sigma}\right]\right\} \quad (52)$$

$$\dot{x} = -g/b_3 - \left(1 + i\right)\sigma^{\frac{i+1}{2}} \times \\ \times\left\{C_1'' J_{1+i}\left[2\sqrt{b_3\sigma}\right] + C_2'' Y_{1+i}\left[2\sqrt{b_3\sigma}\right]\right\} - \\ -\sqrt{b_3}\sigma^{\frac{i}{2}}\left\{C_1'' J_{2+i}\left[2\sqrt{b_3\sigma}\right] + C_2'' Y_{2+i}\left[2\sqrt{b_3\sigma}\right]\right\} \quad (53)$$

Shovel movement during third stage, expressed using (38) and (39) at $t = t_3$, and $\omega_0 > \vartheta$

$$x_3 = (m_1 + \mu)g/k + Me^{-\vartheta t_3} sin(\omega t_3 + N) \quad (54)$$

$$\dot{x}_3 = -M\vartheta e^{-\vartheta t_3} sin(\omega t_3 + N) + M\omega e^{-\vartheta t_3} cos(\omega t_3 + N) \quad (55)$$

Accordingly, at $t = t_3$ and $\omega_0 < \vartheta$ Eqs. (41), (42) give

$$x_3 = (m_1 + \mu)g/k + e^{-\vartheta t_3}\left(M'e^{\omega t_3} + Ne^{-\omega t_3}\right) \quad (56)$$

$$\dot{x}_3 = M'(\omega - \vartheta)e^{(\omega-\vartheta)t_3} - N'(\omega + \vartheta)e^{-(\omega+\vartheta)t_3} \quad (57)$$

and

$$C_1'' = -\pi\sigma^{\frac{1-i}{2}}\left[\dot{x}_3 + g/b_3 + (x_3 - a_3)(1+i)1/\sigma\right]\times \times Y_{1+i}\left(2\sqrt{b_3\sigma}\right) - \pi\sqrt{b_3}(x_3 - a_3)\sigma^{-i/2}\, Y_{2+i}\left(2\sqrt{b_3\sigma}\right) \quad (58)$$

$$C_2'' = \pi\sigma^{\frac{1-i}{2}}\left[\dot{x}_3 + g/b_3 + (x_3 - a_3)(1+i)1/\sigma\right]\times \times J_{1+i}\left(2\sqrt{b_3\sigma}\right) + \pi\sqrt{b_3}(x_3 - a_3)\sigma^{-i/2}\, J_{2+i}\left(2\sqrt{b_3\sigma}\right) \quad (59)$$

In the fifth step the discharge stops (pause after discharge). Differential equation, which describes movementin this case [15]

$$m_1\ddot{x} + c\dot{x} + kx = m_1 g \quad (60)$$

If $\Omega = \sqrt{k/m_1}$, $\Theta = c/2m$ and $\Omega > \Theta$ solution of the Eq. (60) according to [15] shall be

$$x = m_1 g/k + Ae^{-\Theta t} sin(pt + B) \quad (61)$$

$$\dot{x} = -A\Theta e^{-\Theta t} sin(pt + B) + Ape^{-\Theta t} cos(pt + B) \quad (62)$$

where $p^2 = \Omega^2 - \Theta^2$, A and B is constants. At $t = 0$, $x = x_4$, $\dot{x} = \dot{x}_4$

$$\left.\begin{aligned} A^2 &= (x_4 - m_1 g/k)^2 + \left[\dot{x}_4 + \Theta(x_4 - m_1 g/k)\right]^2 1/p^2 \\ B &= arctg\frac{x_4 p - m_1 gp/k}{\dot{x}_4 + \Theta(x_4 - m_1 p/k)} \end{aligned}\right\} \quad (63)$$

When $\Omega < \Theta$ solution of Eq. (60) according to [15] shall be

$$x = m_1 g/k + A'e^{(p-\Theta)t} + B'e^{-(p+\Theta)t} \quad (64)$$

where $p^2 = \Theta^2 - \Omega^2$, A' and B' is constant.

At $t = 0$, $x = x_4$, $\dot{x} = \dot{x}_4$

$$\left.\begin{aligned} A' &= \dot{x}_4/2p + (1/2 + \Theta/2p)(x_4 - m_1 g/k) \\ B' &= -\dot{x}_4/2p + (1/2 - \Theta/2p)(x_4 - m_1 g/k) \end{aligned}\right\} \quad (65)$$

Dimensions x_4 and $\dot{x}_4$ are correspondingly the displacement and speed of the shovel at the end of step 4. These values are derived from Eqs. (49), (50) at $t_4 = \mu/\kappa_3$. The developed mathematical model allows for analysis and investigation of weight batchers with development stages. Interaction and influence of various system elements on dosing process thus can be evaluated making the batcher design predictable.

4. Conclusions

A typical structural scheme of weight batcher has been analyzed by using physical model comprising sensitive element regarded as elastic system with one degree of freedom with the attached body having variable mass.

Dynamic mathematical model of the batcher has been developed, which describes the batcher performance at its five key working stages. The working stages covered are: filling with the product at increased productivity, accurate low-rate top-up filling, filling suspension (pause before discharge), product discharge and discharge suspension (pause after discharge). Mathematical model allows for dosing process analysis of weight batchers considering influence of its key elements, their interaction and can be used as a working tool in various stages of bathers' development and design.

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BIRIŲJŲ PRODUKTŲ SVORINIO DOZATORIAUS DARBO MATEMATINIS MODELIS

R e z i u m ė

Straipsnyje pateikta biriųjų produktų svorinio dozavimo tipinė struktūrine schema ir dozavimo proceso fizinis modelis, kaip vieno laisvės laipsnio tampri sistema, kurią veikia kintamos masės kūnas. Pateiktas svorinio dozatoriaus darbo per visą ciklą matematinis modelis. Svorinio dozatoriaus darbo ciklą sudaro penki etapai. Pirmasis etapas - tai produkto tiekimas padidintu našumu, antrasis - produkto tiekimas mažu našumu, trečiasis – kai produkto tiekimas sustabdytas (pauzė prieš išpylimą). Ketvirtasis - produkto išpylimas. Penktasis – kai produkto išpylimas baigtas (pauzė po išpylimo). Šis svorinio dozatoriaus darbo matematinis modelis leidžia teoriškai nagrinėti dozatoriaus darbo procesą ir dozatoriaus schemos bei konstrukcijos ypatumus.

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MATHEMATICAL MODEL OF THE OPERATION OF A WEIGHT BATCHER FOR DRY PRODUCTS

S u m m a r y

The article presents a typical structural scheme and physical model of weight bather for dry products regarded as an elastic system with one degree of freedom, operating together with the attached variable mass body. Dynamic mathematical model of the batcher has been developed. The model describes weight batcher's performance at its five key working stages: filling with the product at increased productivity, accurate low-rate top-up filling, filling suspension (pause before discharge), product discharge and discharge suspension (pause after discharge). Mathematical model allows for theoretical investigation of weight bather performance throughout its entire working cycle as well as analysis of its structural and design features.

Received November 10, 2010
Accepted September 09, 2011