

# Parameter sensitivity analysis of axial vibration for lead-screw feed drives with time-varying framework

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## 1. Introduction

With the rapid development of the manufacturing industry, there is an urgent demand for the high efficiency and high precision of CNC machine tools. However, the improvement of the efficiency and the precision is often restricted by the structure of the machine tools [1]. The high speed machining (HSM) requires machine tool feed drive systems to achieve a great feedrate in a limited travel distance with a huge acceleration [2].

Ball screw feed drive systems that are widely used in CNC machine tools transform rotary motion of motor into rectilinear motion of table (or cutter) utilizing the complicated screw-ball-nut transmission structure. In the course of the motion transformation, slender screw may be induced torsional, axial, and bending deformations and vibrations. Especially in high speed and high acceleration, the dynamic behavior of the transmission framework will depress the machining quality and efficiency of the CNC machine tools, which is a new challenge for conventional design and control theories and methods. Therefore, many researchers have devoted their efforts to the research into the dynamics of the ball screw feed drives.

For example, Poinet et al. [3] considered the feed drive model with lumped springs and lumped masses, where the model consisting of 8 masses and 7 springs (8M7S) were presented as a transmission axis example. Chen et al. [4] built the dynamic model of the ball screw feed drives using the Lagrange method and proposed a topology structure optimization method to reduce the moving weight. Erkorkmaz et al. [5, 6] built and identified the dynamic model of the ball screw feed drives, and compensated for the torsional vibrations and axial vibrations with notch filter and sliding mode controller respectively. Zaeh and Oertli [7] established the whole FEM model of ball screw feed drive systems and simulated the axis control system. Varanasi and Nayfeh [8] developed a low-order dynamics model of lead-screw system that accounts for the distributed inertia of the screw, the compliance, damping of the thrust bearings, nut, and coupling. Whalley and Ebrahimi et al. [9] used distributed-lumped parameter techniques for the dynamic analysis of machine tool systems. Uchiyama [10] presented a new controller design for feed drive systems with adaptive feed-forward and feedback controllers according to a fourth-order dynamic model which considers the compliance of the lead-screw drive. Okwudire and Altintas [11] presented a hybrid finite element model of the screw-nut interface, which includes the effects of the coupled lateral, torsional, and axial deformations.

However, the present dynamic investigations of

the feed drive systems are usually on the status to select the practical moving (changeable position) table in a specified fixed position, such as the middle of ball screw, which could not reflect the real time dynamic characteristics of the drives during the machining process. Actually, the table position is continuously changeable and the workpiece often has high metal removal quantity in the machining process. The researches indicate that the time-varying table position has a distinct influence on the torsional dynamics of the lead-screw feed drives [12].

Here, we give a sensitivity analysis of the axis vibration characteristic giving a full consideration of the time-varying position of moving table and the changeable workpiece mass in the lead-screw feed drive system on the basis of the finite element and lumped parameter methods. In the following section, the axial dynamic model of feed drive framework is built and the main formulae employed in the theoretical model are introduced. The predictions of the first three natural frequencies and vibration modes of the axial vibration are shown in Section 3. Finally, some conclusions of this research are remarked in Section 4.

## 2. Axial dynamic model of lead-screw feed drive with time-varying framework

The sketch of axial dynamic model of the machine tool rotor-screw system is shown in Fig. 1. To analyze the dynamic characteristic of the mechanical system, here, finite element and lumped parameter models are developed, where the rotor-screw system is divided into some axial elements additional to the coupling ones of input component and output component. For the in-time change of table and nut, we consider them as lumped masses and the effects of the contact stiffness between the table and guide rails are neglected. Moreover, the axial stiffness of each bearing is regarded as lumped stiffness.

In Fig. 1,  $k_{rf}$  is the axial stiffness of the front rotor bearing;  $k_{rb}$  is the axial stiffness of the back rotor bearing;

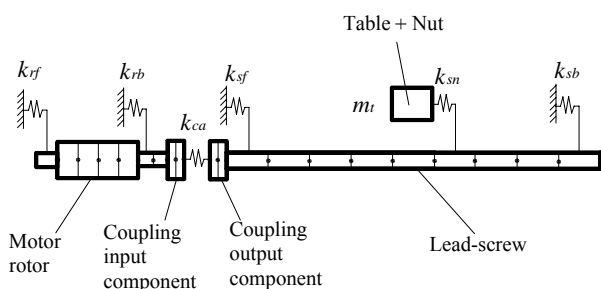


Fig. 1 Sketch of machine tool rotor-screw system physical model

$k_{ca}$  is the axial stiffness between the coupling in put component and output component;  $k_{sf}$  is the axial stiffness of the front screw bearing;  $k_{sb}$  is the axial stiffness of the back screw bearing;  $k_{sn}$  is the axial stiffness between the nut and the screw, and  $m_i$  is the aggregate mass of the table and the nut, which will include the mass of the workpiece and the clamp if they are considered.

### 2.1. Element matrixes of mass and stiffness

Divide the rotor-screw into  $n$  equal elements along its length. Assume that each element has length of  $l$ . Denote the axial displacement by  $u(x,t)$ . Then we have the node unknowns at each element being  $u_i(t)$  and  $u_{i+1}(t)$  (Fig. 2). Here the subscript  $i$  indicates the node number. By means of the linear interpolation, we can get the formula to express the axial displacement inside the element relevant to the node displacements in the form

$$u(x,t) = \sum_{j=i}^{i+1} N_j(x)u_j(t) \quad (1)$$

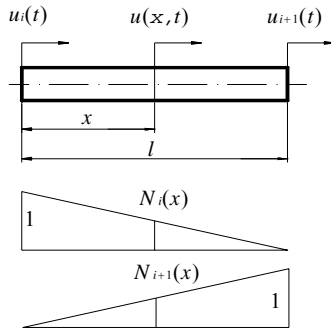


Fig. 2 A axial element in FEM

where  $N_i(x)$  and  $N_{i+1}(x)$  are the shape functions as shown in Fig. 2, and they are explicitly formulated by

$$N_i(x) = 1 - \frac{x}{l}, \quad N_{i+1}(x) = \frac{x}{l} \quad (2)$$

For simplicity, we compact Eq. (1) by the following matrix form

$$u(x,t) = \mathbf{N}(x)\mathbf{u}_i(t) \quad (3)$$

In the form,  $\mathbf{N}(x) = [1-x/l, x/l]$ ,  $\mathbf{u}_i(t) = [u_i(t), u_{i+1}(t)]^T$ , where the superscript  $T$  represents the transpose of a matrix or column.

Let the mass per unit length of the axial element be  $m_l$ . Then, the kinetic energy of the element  $i$  at instant  $t$  can be expressed by

$$T_i(t) = \frac{1}{2} \int_0^l m_l \left[ \frac{\partial u(x,t)}{\partial t} \right]^2 dx = \frac{1}{2} \dot{\mathbf{u}}_i^T(t) \mathbf{M}_i \dot{\mathbf{u}}_i(t) \quad (4)$$

here  $\mathbf{M}_i$  is the element mass matrix that is formulated by

$$\mathbf{M}_i = \int_0^l m_l \mathbf{N}^T(x) \mathbf{N}(x) dx = \frac{m_l l}{6} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad (5)$$

Denote the elastic modulus of the axial element

by  $E$ , and its cross-sectional area by  $A$ . Then, the deformed potential energy in the element at instant  $t$  can be calculated by

$$V_i(t) = \frac{1}{2} \int_0^l EA \left[ \frac{\partial u(x,t)}{\partial x} \right]^2 dx = \frac{1}{2} \mathbf{u}_i^T(t) \mathbf{K}_i \mathbf{u}_i(t) \quad (6)$$

in which  $\mathbf{K}_i$  is the element stiffness matrix that is formulated as

$$\mathbf{K}_i = \int_0^l EA \frac{d\mathbf{N}^T(x)}{dx} \frac{d\mathbf{N}(x)}{dx} dx = \frac{EA}{l} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad (7)$$

For the element that joins the bearing, the bearing can be simplified the axial spring that is applied to the middle of the element (Fig. 3).

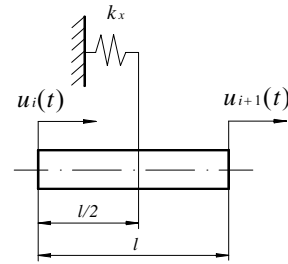


Fig. 3 Element that joins the bearing

Let the stiffness of the spring be  $k_x$ . Then, the deformed potential energy in the element at instant  $t$  can be calculated by

$$V_i'(t) = \frac{1}{2} \mathbf{u}_i^T(t) (\mathbf{K}_i + \mathbf{K}_x) \mathbf{u}_i(t) \quad (8)$$

where

$$\mathbf{K}_x = \frac{k_x}{4} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad (9)$$

### 2.2. System dynamic equations

Integrating the analytic result of each element, and assembling the element mass matrix and the element stiffness matrix, respectively, one can get the whole mass matrix  $\mathbf{M}$  and the whole stiffness matrix  $\mathbf{K}$ .

Here, the axial stiffness of the coupling is considered as lumped stiffness  $k_{ca}$ , then, the assembling method is as following. The two elements joint by  $k_{ca}$  can form an element subsystem as shown in Fig. 4. Thus, the deformed potential energy in the subsystem at instant  $t$  can be calculated by

$$V_s(t) = \frac{1}{2} \mathbf{u}_i^T(t) \mathbf{K}_i \mathbf{u}_i(t) + \frac{1}{2} \mathbf{u}_{i+2}^T(t) \mathbf{K}_{i+2} \mathbf{u}_{i+2}(t) + \frac{1}{2} k_{ca} (u_{i+1}(t) - u_{i+2}(t))^2 \quad (10)$$

Denote the displacement column vector by  $\mathbf{u}_i'(t) = [u_i(t), u_{i+1}(t), u_{i+2}(t), u_{i+3}(t)]^T$ , then, Eq. (10) can be rewritten by

$$V_s(t) = \frac{1}{2} \mathbf{u}_i'^T(t) \mathbf{K}' \mathbf{u}_i'(t) \quad (11)$$

where

$$\mathbf{K}' = \begin{bmatrix} \frac{EA}{l} & -\frac{EA}{l} & 0 & 0 \\ -\frac{EA}{l} & \frac{EA}{l} + k_{ca} & -k_{ca} & 0 \\ 0 & -k_{ca} & \frac{EA}{l} + k_{ca} & -\frac{EA}{l} \\ 0 & 0 & -\frac{EA}{l} & \frac{EA}{l} \end{bmatrix} \quad (12)$$

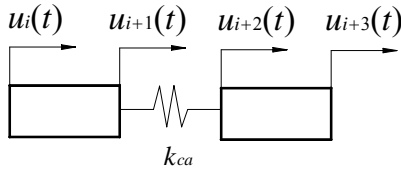


Fig. 4 Element subsystem with lumped stiffness of coupling

The lumped mass  $m_i$  is adopted for the table and the nut, and the lumped stiffness  $k_{sn}$  is regarded as the axial stiffness between the nut and the screw. Then, the lumped mass  $m_i$  and the screw element joint by  $k_{sn}$  can form an element subsystem as shown in Fig. 5.

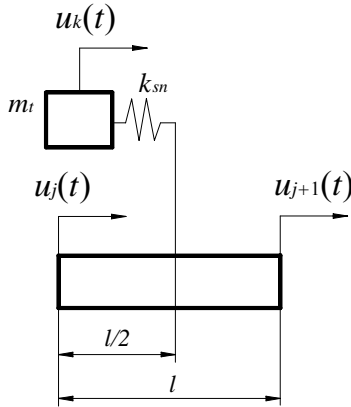


Fig. 5 Element subsystem with lumped parameters of the table and the nut

Let the number of the screw element that connect with the nut be  $j$  and the number of the element of the table and the nut be  $k$ . Then, the kinetic energy of the subsystem at instant  $t$  can be expressed by

$$T_i(t) = \frac{1}{2} \dot{\mathbf{u}}_j'^T(t) \mathbf{M}_j \dot{\mathbf{u}}_j'(t) + \frac{1}{2} m_i (\dot{u}_k(t))^2 \quad (13)$$

Denote the displacement column vector by  $\mathbf{u}_j'(t) = [u_j(t), u_{j+1}(t), u_k(t)]^T$ , then, Eq. (13) can be rewritten by

$$T_i(t) = \frac{1}{2} \dot{\mathbf{u}}_j'^T(t) \mathbf{M}'_j \dot{\mathbf{u}}_j'(t) \quad (14)$$

in which

$$\mathbf{M}'_j = \begin{bmatrix} \frac{m_i l}{3} & \frac{m_i l}{6} & 0 \\ \frac{m_i l}{6} & \frac{m_i l}{3} & 0 \\ 0 & 0 & m_i \end{bmatrix} \quad (15)$$

The deformed potential energy in the subsystem at instant  $t$  can be calculated by

$$V_s(t) = \frac{1}{2} \mathbf{u}_j'^T(t) \mathbf{K}_j \mathbf{u}_j'(t) + \frac{1}{2} k_{sn} \left( \frac{u_i(t) + u_{i+1}(t)}{2} - u_k(t) \right)^2 = \frac{1}{2} \mathbf{u}_j'^T(t) \mathbf{K}'_j \mathbf{u}_j'(t) \quad (16)$$

where

$$\mathbf{K}'_j = \begin{bmatrix} \frac{EA}{l} + \frac{k_{sn}}{4} & -\frac{EA}{l} + \frac{k_{sn}}{4} & -\frac{k_{sn}}{2} \\ -\frac{EA}{l} + \frac{k_{sn}}{4} & \frac{EA}{l} + \frac{k_{sn}}{4} & -\frac{k_{sn}}{2} \\ -\frac{k_{sn}}{2} & -\frac{k_{sn}}{2} & k_{sn} \end{bmatrix} \quad (17)$$

In the actual calculation, the number of the table and the nut is often taken as the last number. Thus, the mass matrix and the stiffness matrix should be assembled in the corresponding row and column.

Take the node displacements as the generalized coordinates  $q_i$ , and the corresponding generalized forces as  $Q_i$ , where  $i = 1, 2, \dots, n$  ( $n$  is the amount of the independent nodes considering the boundary conditions of the dynamic system). Then, by means of the FEM using the Lagrange dynamic equation, we can get the solvable system dynamic equations in the matrix form

$$\mathbf{M} \ddot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{Q} \quad (18)$$

in which  $\mathbf{q} = [q_1, q_2, \dots, q_n]^T$ ,  $\mathbf{Q} = [Q_1, Q_2, \dots, Q_n]^T$ .

For the analysis of free vibration of the system, we have  $\mathbf{Q} = 0$ . Thus, the system dynamic equations can be expressed by

$$\mathbf{M} \ddot{\mathbf{q}} + \mathbf{K} \mathbf{q} = 0 \quad (19)$$

The position of translation components and the workpiece mass vary with time in machining process, and, especially in the manufacture of aircraft components, it is common for the workpiece mass to change substantially [12]. Therefore, the global mass matrix and the global stiffness matrix change with time too, i.e.,  $\mathbf{M} = \mathbf{M}(t)$  and  $\mathbf{K} = \mathbf{K}(t)$ , which will result in the change of natural frequencies and modes in the dynamic system with time too.

### 3. Numerical results and discussions (case study)

In this section, we display some results of the numerical analysis proposed in previous section. An axial dynamic finite element analysis program for lead-screw feed drive is written in the MATLAB programming lan-

guage. To show the effect of changeable position of translation components on the dynamic characteristic of the system when the time-varying position is taken into account, here, a case study is employed. Table 1 lists the geometrical parameters of the X-axis feed drive system of a machine tool and Table 2 lists the lumped parameters that are considered.

For the free vibration analysis to the dynamic system at the following, first of all, let us consider that the motion travel or position  $x$  of the table is specified from  $0.15L$  to  $0.85L$  (here,  $L$  is the length of lead-screw) and the workpiece mass is changed from 700 to 200 kg. Fig. 6 exhibits the characteristic curves of the first three natural frequencies of the dynamic system varying with the table position and the workpiece mass. When the workpiece mass is 200kg and the table position is  $0.15L$ ,  $0.5L$  and  $0.85L$ , the mode shapes of the first three orders of vibration are shown in Fig. 7.

Table 1

Geometrical parameters of the X-axis feed drive system

	Length, mm	Outer diameter, mm	Inner diameter, mm	Lead, mm
Rotor	220	150	0	/
Motor shaft	120	40	0	/
Coupling	101	104	40	/
Lead-screw	3500	50	0	20

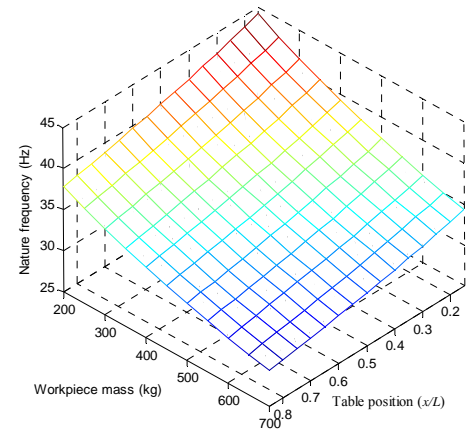
Table 2

Lumped parameters that are considered

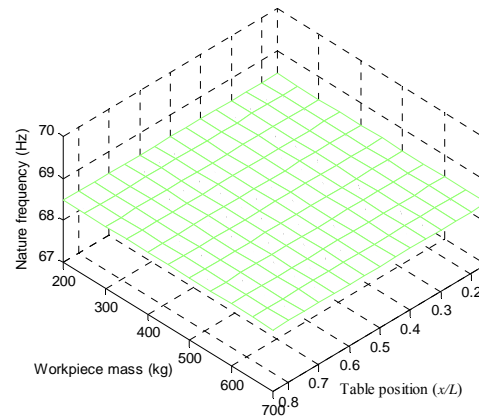
	Axial stiffness, N/m	Mass, kg
Front rotor bearing	$3.7 \times 10^6$	/
Back rotor bearing	$1.9 \times 10^6$	/
Front screw bearing	$1.5 \times 10^9$	/
Back screw bearing	$2.8 \times 10^6$	/
Coupling	$8.0 \times 10^5$	/
Table + Nut	$6.3 \times 10^7$	550

In Fig. 6 the natural frequency of the first mode varies in the range of [29.26 Hz, 44.26 Hz] and the variation relates to both the table position and workpiece mass; the natural frequency of the second mode keeps an invariant value 68.47 Hz; the natural frequency of the third mode varies in the range of [387.58 Hz, 455.77 Hz] and the variation mainly relates to the table position. The results indicate that the natural frequency of the first mode is sensitive to both changeable table position and workpiece mass to a certain extent, and that the natural frequency of the second mode is insensitive to them, and the natural frequency of the third mode is sensitive to the table position whereas insensitive to workpiece mass.

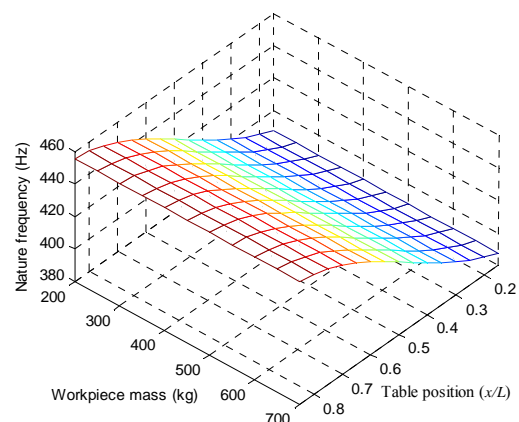
According to Fig. 7, one can find that the relative amplitude of the table is maximal in the first mode, the motor rotor in the second mode and the back end of the screw in the third mode, which indicates that the first mode is focused on table vibration, the second mode on rotor vibration and the third mode on screw vibration.



a



b



c

Fig. 6 The characteristic curves of the first three natural frequencies vary with the table position and the reduced workpiece mass: a - first mode; b - second mode; c - third mode

Moreover, considering that the axial stiffness  $k_{sb}$  of the back bearing of the screw is enhanced, we can get the curves of the first three natural frequencies varying with the table position for the different  $k_{sb}$ , as shown in Fig. 8. From Fig. 8, it is found that the enhancement of the axial stiffness of the back bearing can effectively depress the sensitivity of the first nature frequency on the changeable table position; the second nature frequency is invariant on the whole; the third nature frequency increases with enhancement of the axial stiffness of the back bearing, but the trend of the curves has little change.

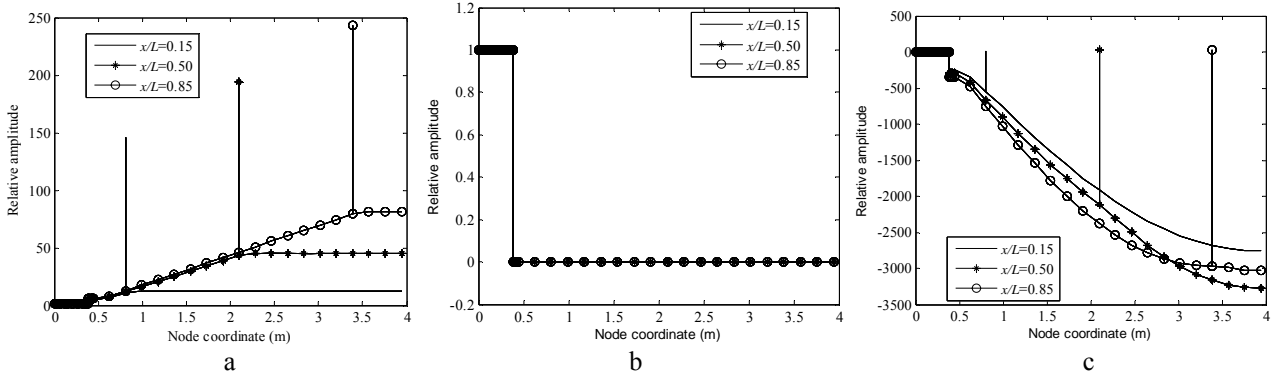


Fig. 7 The mode shapes of the first three orders of free vibration when the table is located different position of the lead-screw: a - first mode; b - second mode; c - third mode

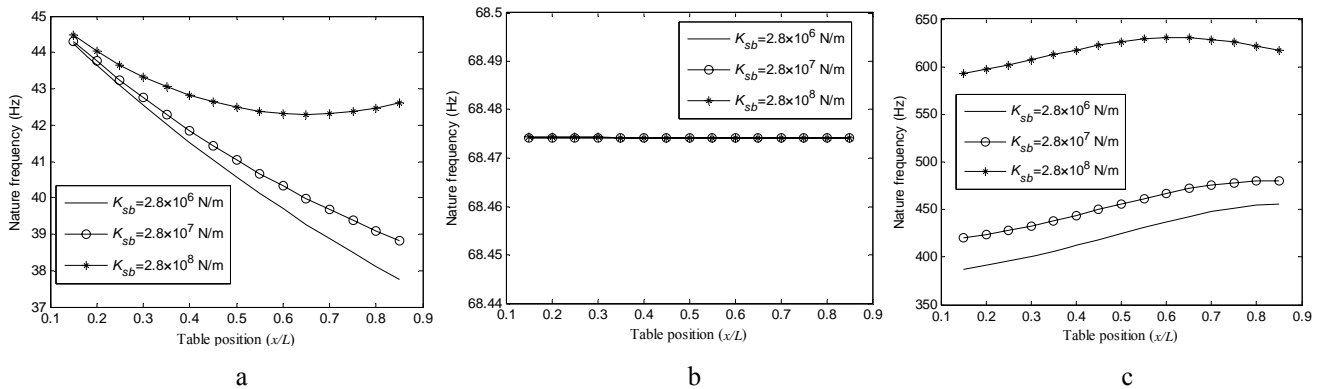


Fig. 8 The curves of the first three natural frequencies vary with the table position for the different  $k_{sb}$ : a - first mode; b - second mode; c - third mode

#### 4. Conclusion

1. An axial dynamic model of lead-screw feed drive with time-varying framework is built by using the finite element and lumped-parameter methods.

2. A numerical analysis to the axial vibration of lead-screw feed drive system is presented to get an insight into the effect of in-time moving position of the table and changeable workpiece mass on the free vibration characteristic of the system. The results obtained demonstrate that the natural frequency of the first mode focusing on table vibration is sensitive to both changeable table position and workpiece mass to a certain extent; the natural frequency of the second mode focusing on rotor vibration is unsensible to changeable table position and workpiece mass; the natural frequency of the third mode focusing on screw vibration is sensitive to the table position whereas unsensible to the workpiece mass.

3. Enhancing the axial stiffness of the back screw bearing was analyzed, which indicates that the enhancement of the axial stiffness of the back screw bearing can effectively depress the sensitivity of the first nature frequency on the changeable table position.

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PASTŪMOS SRAIGTO SU LAIKE KINTANČIA  
POZICIJA AŠINIŲ VIBRACIJŲ PARAMETRŲ  
JAUTRUMO ANALIZĖ

Re z i u m ė

Straipsnyje pateiktas nuosavų ašinių svyravimų dinaminis tyrimas rotoriaus – jėgos sraigto sistemai. Naudojant baigtinius elementus, sukurtas rotoriaus - jėgos sraigto dinaminis modelis ir analizuota kintamos stalo padėties bei darbinės masės įtaka pirmosioms trimis ašinėms nuosavų svyravimų *modoms*. Rezultatai rodo, kad pirmas nuosavasis dažnis atspindi stalo svyravimus, yra iki tam tikro dydžio jautrus stalo padėčiai ir detalės masei. Antrasis *modos* nuosavas dažnis atspindi rotoriaus svyravimą ir yra nejautrus stalo padėčiai. Trečiasis *modos* nuosavas

dažnis atspindi sraigto svyravimus ir yra jautrus stalo padėčiai, bet nejautrus detalių masei. Antroje tyrimo dalyje buvo didintas sraigto galinio guolio standumas. Gauti rezultatai parodė, kad sraigto galinio guolio standumo didinimas efektyviai sumažino pirmosios modos nuosavųjų svyravimų jautrumą stalo padėties kitimui.

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PARAMETER SENSITIVITY ANALYSIS OF AXIAL  
VIBRATION FOR LEAD-SCREW FEED DRIVES  
WITH TIME-VARYING FRAMEWORK

S u m m a r y

This paper establishes a dynamic model of lead-screw feed drives by means of the finite element and lumped-parameter and analyzes the effects of changeable table position and workpiece mass on the first three axial modes of the free vibration. The results indicate that the natural frequency of the first mode focusing on table vibration is sensitive to both changeable table position and workpiece mass to a certain extent; the natural frequency of the second mode focusing on rotor vibration is unsensible to changeable table position and workpiece mass; the natural frequency of the third mode focusing on screw vibration is sensitive to the table position whereas unsensible to the workpiece mass. In the second stage of research, the results display that the enhancement of the axial stiffness of the back screw bearing can effectively reduce the sensitivity of the first nature frequency on the changeable table position.

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