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Buckling analysis of composite long cylinders using probabilistic finite element method

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1. Introduction

Fibre-reinforced composite materials are increasingly used in marine application over the past few decades due to their light weight and high resistance to salt water corrosion [1]. There materials are now being applied for composite pressure hulls, autonomous underwater vehicle, and even offshore oil equipment [2-7], which usually subject to high pressure of seawater. For these structures, external hydrostatic pressure-induced buckling and crushing tends to dominate structural performance.

In recent years, composite structures, especially of cylinders, subjected to external hydrostatic pressure have been widely studied by using various methods, such as the mathematical formulation method and finite element method. Hur et al. [8, 9] studied the buckling and postbuckling behaviours of composite cylinders by performing hydrostatic external pressure tests. Three finite element analysis programs including ACOS, MSC.NASTRAN and MSC.MARC were used for the failure analysis. It was identified that the results of finite element analysis and the hydrostatic test indicated good matches. Frulloni et al. [10] studied the failure behaviour of lattice composite hollow structures subjected to external hydrostatic pressure. In order to understand the failure mechanism, finite element analysis was used to evaluate the buckling pressure of the tested tubes by using ANSYS software. Tafreshi [11, 12] proposed a computational modelling of delamination buckling and postbuckling of laminated composite cylindrical shells subjected to external pressure or combined axial compression and external pressure. The finite element analysis which was carried out using ABAQUS verified the computational results. Graham [13, 14] developed the analytical model which were developed in conjunction with a series of model tests and used in the design of the large scale composite hull model. The results were validated by the finite element analysis with ABAQUS software.

From these literatures, it can be seen that the finite element analysis method has been widely used to study the composite pressure vessel. Various commercial software, for example ANSYS [10], ABAQUS [11-14, 15], ADINA [16] and SAMCEF [17], are used to perform the finite element analysis. However, these analyses are mainly deterministic. The reliability of composite pressure vessel does not be evaluated by using finite element analysis method. To address the growing need for stochastic and probabilistic finite element analysis, ANSYS Inc. released the ANSYS Probabilistic Design System (PDS) [18]. It can be used for an uncertainty analysis or a reliability analysis. The PDS includes both of the Monte Carlo Simulation (MCS) method and Response Surface Method (RSM). The method has been used to study the probabilistic problems for various structures. Nakamura et al. [19] demonstrated the probabilistic thermal analysis of an atmospheric re-entry vehicle structure and investigated the probabilistic temperature response by using MCS. Zulkifli et al. [20] evaluated the reliability or fatigue life of the solder joints in the ball grid array package by using RSM. Nemeth et al. [21] studied the effect of specimen dimension of single crystal SiC on the strength response by using PDS. Liu et al. [22] studied the strength reliability of composite laminated high pressure hydrogen storage vessel by using MCS and RSM.

This work aims to study the effects of uncertainties of material properties and physical dimensions on the critical buckling pressure of composite long cylinders subjected to external hydrostatic pressure by using probabilistic finite element analysis method. The longitudinal modulus, transversal modulus, shear modulus, Poisson's ratio of composite material, unsupported length, inside radius, thickness and winding angles of inner and outer layers of composite long cylinders are taken as random input parameters, and the critical buckling pressure is taken as random output response. A total of four carbon-epoxy composite long cylinders were fabricated and tested in a hyperbaric testing chamber to validate the finite element analysis results.

2. Finite element analysis

2.1. Deterministic analysis

The structural model used to represent the composite long cylinder under study is schematically shown in Fig. 1. The cylindrical wall is composed of 4 inner plies with winding angle of α_i and 13 outer plies with winding angle of α_o with respect to the axis of the cylinder. Each of the plies has equal thickness. The stacking sequence can be denoted by $[\alpha_{i4}/\alpha_{o13}]_T$, as shown in Fig. 1, a. The geometry of the long cylinder is characterized by its overall length L_0, unsupported length L, inside radius R and thickness T, as

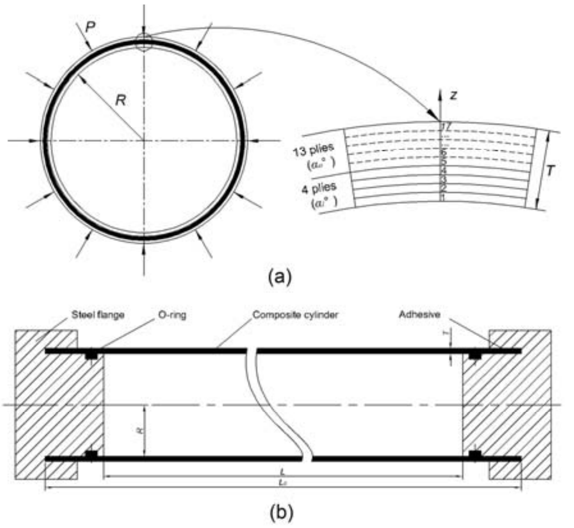


Fig. 1 Structural model of composite long cylinder subjected to external hydrostatic pressure

shown in Fig. 1, b. The dimensions of cylinder are given in Table 1. It is noted that there are four overall lengths and unsupported lengths, which shows that four long cylinders with different lengths are researched in this work. The critical external pressure of composite long cylinder is denoted by *P*.

The longitudinal modulus, transversal modulus, shear modulus, Poisson's ratio are the four most important mechanical properties of composite material, which are also given in Table 1. For the sake of simplicity, the through-thickness modulus is assumed to be same as transversal modulus E_T; the longitudinal and transversal through-thickness shear moduli are assumed to be same as longitudinal in-plane shear modulus G_{LT}; and the minor Poisson's ratio and transversal Poisson's ratio are assumed to be same as major Poisson's ratio ν_{LT}.

Finite element analysis package ANSYS is used to predict the buckling behaviour of composite long cylinders subjected to external hydrostatic pressure. Fibre orientation, thickness distribution, stacking sequence and number of layers are the parameters used to describe the laminated composite structure by using linear layered structural shell element SHELL99. The element allows up to 250 different material layers with different orientations and orthotropic material properties in each layer. It has six degrees of freedom at each node: translations in the nodal *x*, *y*, and *z* directions and rotations about the nodal *x*, *y*, and *z*-axes. Fig. 2, a shows the stacking sequence and material orientation angles used in the long cylinder finite element analysis. It can be found that the orientation angles and winding angles are complementary.

Table 1

Statistical characteristic of dimensions and mechanical properties

Property	Symbol	Mean	COV/Boundary	Distribution type
Longitudinal modulus	E_L, MPa	135000	0.10	Normal
Transversal modulus	E_T , MPa	10000	0.06	Normal
Shear modulus	G_{LT} , MPa	5000	0.08	Normal
Poisson's ratio	ν_{LT}	0.3	0.08	Normal
Overall length	L_0, mm	390/580/680/780	–	–
Unsupported length	L, mm	360/550/650/750	0.01	Normal
Inside radius	R, mm	21.5	0.01	Normal
Thickness	T, mm	1.36	0.02	Normal
Winding angle of inner layers	$\alpha_{i,}$ °	90	±3	Uniform
Winding angle of outer layers	α_o, °	0	±3	Uniform

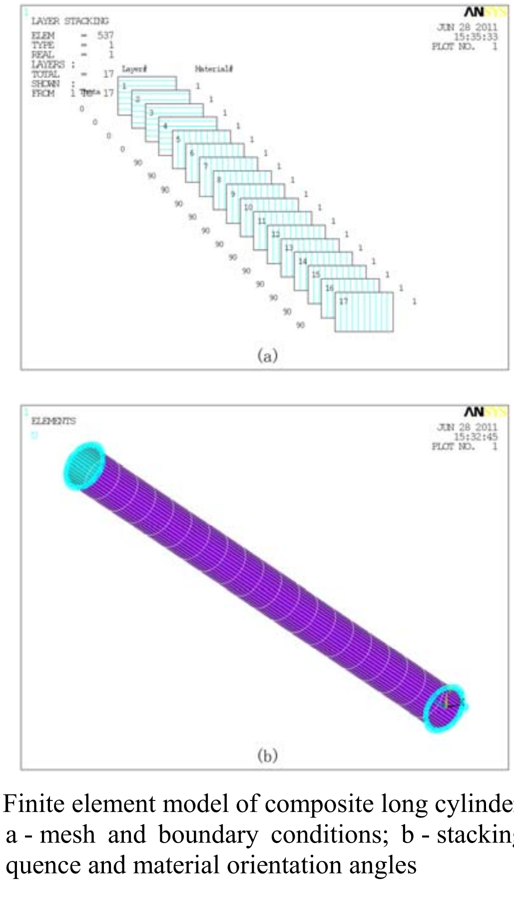


Fig. 2 Finite element model of composite long cylinder: a - mesh and boundary conditions; b - stacking sequence and material orientation angles

The boundary conditions for the finite element analysis are fixed along the both cylinder roots as shown in Fig. 2, b. An external pressure of $P = 1$ is loaded on the external surfaces of composite layers. Static analysis and buckling analysis are performed in sequence in order to obtain the critical buckling pressure.

2.2. Probabilistic analysis

The probabilistic finite element analysis of composite long cylinder is performed by mean of ANSYS/PDS. The PDS is based on the ANSYS parametric design language, which allows users to parametrically build a finite element model, solve it, obtain results and extract characteristic results parameters such as the critical buckling pressure for example. The PDS includes MCS and RSM. The MCS does not make any simplification or assumptions in the deterministic of probabilistic model, and the required number of simulations is not a function of the number of input variables, whereas this method requires plenty of computational time. The RSM replaces the true input-output relationship of MCS by an approximation function, and the evaluation of the response surface is much faster than a finite element solution. However, this method is unusable when true input-output relationship is not continuous [18].

In this work, both of MCS and RSM are used to execute the probabilistic analysis of composite long cylinder. The mechanical properties of composite material including longitudinal modulus, transversal modulus, shear modulus and Poisson's ratio, and the dimensions of cylinder including unsupported length, inside radius, thickness and winding angles of inner and outer layers are taken as random input parameters, and the critical buckling pressure is taken as random output response. The statistical characteristics for them including mean, coefficient of variation (COV) and distribution type are given in Table 1. The uncertainties of mechanical properties and physical dimensions are influenced significantly by the manufacturing of composite materials and cylinders. In general, the statistical characteristics are achieved based on the extensive data collection and data analysis. However, in the absence of sufficient and good quality data, professional expertise has to be employed. In this work, the variables values estimated based on tests and engineering judgment are used. For the winding angles, uniform distribution is assumed, and for all the other random variables, normal distribution is assumed.

For the MCS, the Latin Hypercube Sampling is selected due to that this technique avoids repeating samples that have been evaluated, and also forces the tails of a distribution to participate in the sampling process. In this work, 2000 Latin Hypercube loops are run which are sufficient to obtain converged outputs. For the RSM, the central composite design is used to locate the sampling points in the design space. A total of 147 designs of experiment plus 10000 Monte Carlo simulations are run in exploiting the response surface result. Since each analysis iteration takes about 10 s using 2.5 GHz quad-core Intel processor, the entire simulation requires about 5.5 h and 25 min for MCS and RSM, respectively.

3. Experiment

According to the structural configuration and physical dimensions shown in Fig. 1 and Table 1, four composite long cylinders with different lengths were manufactured. All of the cylinders were made of carbon-epoxy prepreg tape. Each ply thickness of the composite material is 0.08 mm. The stacking sequence of $[90_4/0_{13}]_T$ was adopted. (Of course, some other winding angles were also selectable [23].) In order to seal the cylinders from hydrostatic pressure, besides the O-rings, the adhesive Aradite AW106 with Hardener HV953U were used between the composite cylinders and the steel flanges. The measured dimensions and errors of the manufactured composite long cylinders are given in Table 2. The winding angles are difficult to measure, which are not given in this paper.

The manufactured composite long cylinders are shown in Fig. 3, a. The external hydrostatic pressure tests of them were carried out in a hyperbaric testing chamber (Fig. 3, b) in Rongsheng Machinery Manufacture Co., Ltd., Huabei oilfield, Hebei, China. A high pressure pump was used to supply hydrostatic pressure. Each of the manufactured long cylinders (No. 1, No. 2, No. 3 and No. 4) was submerged in water for testing consecutively, starting from No. 1 and ending in No. 4. The applied external pressure was increased by 0.01 MPa step by step, and in each step the pressure maintained 5 seconds, till buckling and subsequent collapse behaviours occurred.

Table 2

Dimensions of the manufactured composite long cylinders

Cylinder	Overall length L_0, mm	Error, %	Unsupported length L, mm	Error, %	Inside radius R, mm	Error, %	Thickness T, mm	Error, %
No. 1	392.01	0.52	361.24	0.34	21.48	-0.09	1.34	-1.47
No. 2	580.85	0.15	551.75	0.32	21.78	1.30	1.38	1.47
No. 3	678.24	-0.26	649.52	-0.07	21.85	1.63	1.35	-0.74
No. 4	779.51	-0.06	752.02	0.27	21.32	-0.84	1.39	2.21

a b

Fig. 3 a - composite long cylinder; b - hyperbaric testing chamber for external hydrostatic pressure test

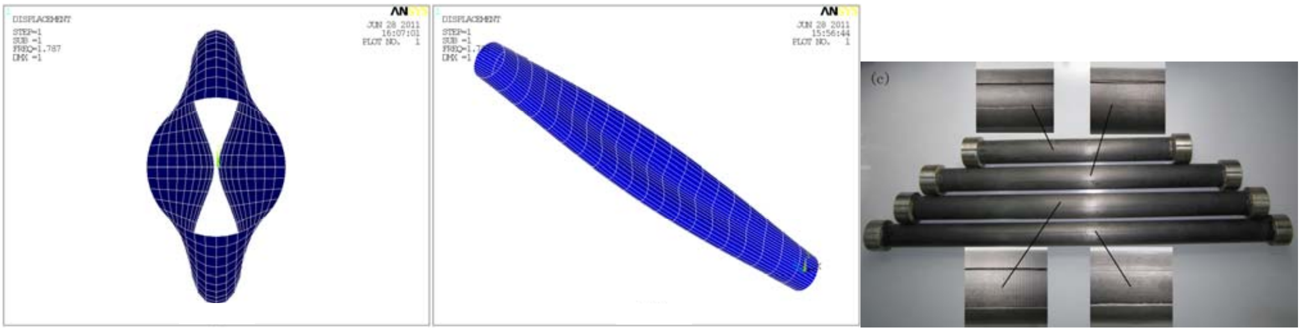
4. Results and discussions

4.1. Comparison of analysis and experimental results

Fig. 4, a and b shows the amplificative buckling mode shape with scale factor of 20 for the composite long cylinder No. 2 with unsupported length of 550 mm according to the finite element method. It is obvious that a two circumferential lobe mode is present. Similarly, for the other three cylinders, two circumferential lobe modes are also observed.

In general, when the buckling and subsequent collapse behaviours occur, the applied pressure drops sharply. In this work, it was found that it was different to distinguish buckling pressure and collapse pressure; therefore, the two pressures are considered as uniform, that is, the long cylinder collapsed immediately after the buckling behaviour occurred. In addition, a louder sound was heard when the long cylinder burst. The experimental failures of composite long cylinders are shown in Fig. 4, c. It can be seen that longitudinal crack is the main failure mode, and in each long cylinder there are two long fracture lines. The experimental buckling mode shapes are similar to the predicted shapes but are slightly different due to the collapse behaviour.

Table 3 gives the predicted and experimental critical buckling (collapse) pressure. It can be seen that with the increasing of unsupported length of composite long cylinders, the buckling pressure decreases slightly, for both of the finite element analyses and hydrostatic pressure tests. The predicted bucking pressures for the four cylinders by using MCS and RSM are almost the same, which are a little lower than the deterministic results. The deviations predicted by deterministic and probabilistic finite element analyses are less than 10% in comparison with the experimental results. For the composite long cylinder No.2, the error is even less than 1%. This shows that the finite element analysis and experimental results show a good agreement.



a b c

Fig. 4 a and b - predicted buckling mode cylinder (No.2); c - experimental buckling modes of composite long cylinders

Table 3

Predicted and experimental critical buckling pressure of composite long cylinders

Cylinder	Experimental Pcr, MPa	Deterministic Pcr, MPa	Error, %	MCS Pcr, MPa	Error, %	RSM Pcr, MPa	Error, %
No. 1	1.81	1.906	5.30	1.9014	5.05	1.9011	5.03
No. 2	1.78	1.787	0.39	1.7850	0.28	1.7846	0.26
No. 3	1.69	1.775	5.03	1.7735	4.94	1.7734	4.93
No. 4	1.61	1.770	9.94	1.7660	9.69	1.7668	9.74

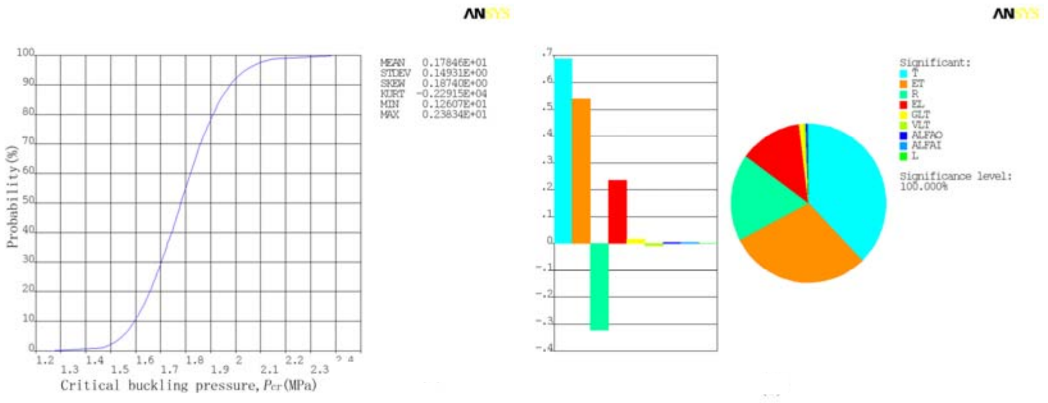
4.2. Probabilistic analysis results

The cumulative distribution function of critical buckling pressure with 95% confidence limit for composite long cylinder No.2 by using RSM is shown in Fig. 5, a. The value of cumulative distribution function at each point states the probability that the related parameter lays under the point. Therefore, when the external pressure is 1.78 MPa, the probability of buckling is 49.79%, which indicates that the buckling behaviour of composite long cylinder highly likely occurs.

Fig. 5, b shows the sensitivity of critical buckling pressure to random input variables. It can be seen that the thickness of composite layers, transversal modulus, inside radius and longitudinal modulus have significant effects on the performance of composite long cylinders. The four variables are responsible for more than 95% of the effect on the failure probability, with the other five variables together making up for the remaining parts. Therefore, the four parameters should be paid more attention when designing composite long cylinders. They are followed by shear modulus, Poisson's ratio, winding angle of outer layers, winding angle of inner layers and unsupported length ($T > E_T > R > E_L > G_{LT} > \nu_{LT} > \alpha_o > \alpha_i > L$). For each simulation, although the orders of sensitivity for some random input variables are different, they have no significant influences on the critical buckling pressure, which can be ignored.

In order to research the effects of COV (boundary) of random input variables on the sensitivity of critical buckling pressure, computations are made when each of the COV (boundary) is varied between ±40% of the values given in Table 1. The results of composite long cylinder No. 2 by using RSM are given in Fig. 6. It is obvious that the rank-order correlation coefficients of thickness, transversal modulus, inside radius and longitudinal modulus are high, which indicates that the four variables have significant effects on the output response.

It can be seen from Fig. 6, a that with the increasing COV of longitudinal modulus, the correlation coefficient of longitudinal modulus increases rapidly, from 0.15 to 0.35. The correlation coefficients of thickness, transversal modulus and inside radius decrease slightly, whereas the correlation coefficients of other five variables are almost changeless. Similar trends are observed for transversal modulus, inside radius and thickness, as shown in Fig. 6, b, f and g, respectively. For the other five variables, the variation of COV almost has no effects on the correlation coefficients of all of the input variables, as shown in Fig. 6, c, d, e, h and i. It is noted that the critical buckling pressure should be strongly dependent on the winding angel. However, in this work, the uncertainty of winding angle which arises from the geometric imperfections during filament winding process is not so large; therefore, the little change of winding angel causes its little effect on the correlation coefficients, as shown in Fig. 6, h and i.



a b

Fig. 5 a - cumulative distribution functions; b - sensitivity of critical buckling pressure of composite long cylinder No. 2 by using RSM

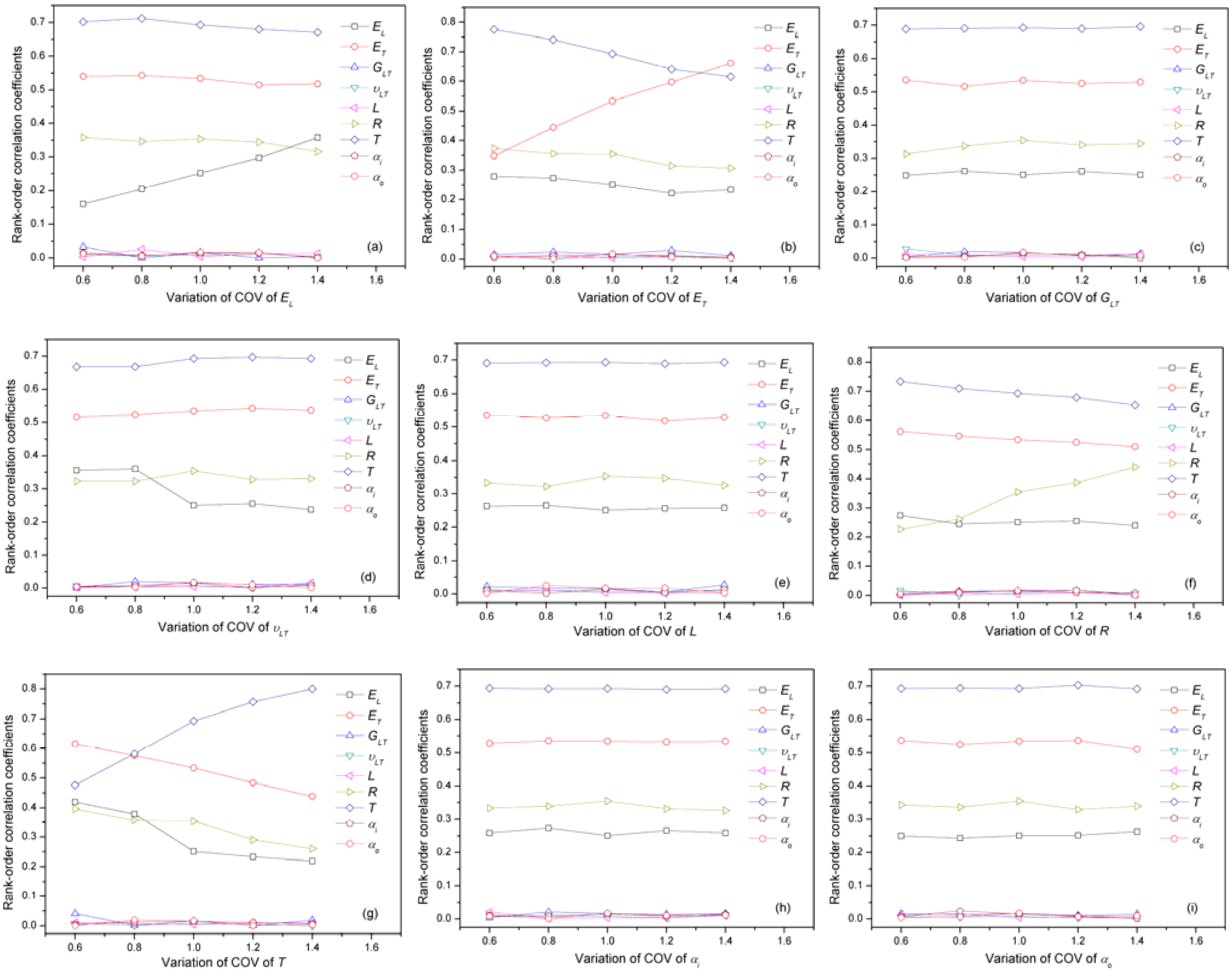


Fig. 6 Effects of COV of: a - longitudinal modulus; b - transversal modulus; c - shear modulus; d - Poisson’s ratio; e - unsupported length; f - inside radius; g - thickness; h - winding angle of inner layers; i - winding angle of outer layers on the sensitivity of critical buckling pressure of composite long cylinder No. 2 by using RSM

5. Conclusions

The buckling behaviours of composite long cylinders subjected to external hydrostatic pressure are researched by using deterministic and probabilistic finite element analyses. The effects of uncertainties of material properties and physical dimensions on the critical buckling pressure are also researched. Four composite long cylinders with different lengths are manufactured. The external hydrostatic pressure tests performed in order to validate the finite element analysis results.

1. The deterministic and probabilistic finite element analyses predict the similar critical buckling pressure, which is a little higher than the experimental results.
2. The probability of buckling predicted by using RSM is correct according to the experimental critical buckling pressure.
3. The thickness of composite layers, transversal modulus, inside radius and longitudinal modulus have significant effects on the performance of composite long cylinders, whereas shear modulus, Poisson’s ratio, winding angle of outer layers, winding angle of inner layers and unsupported length have small influences.
4. The buckling behaviours of finite element analyses by using deterministic and probabilistic methods and hydrostatic pressure tests indicate good matches.

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KOMPOZICINIŲ ILGŲ CILINDRŲ KLUPIMO ANALIZĖ TIKIMYBINIU BAIGTINIŲ ELEMENTŲ METODU

Reziumė

Pažeisto anglies pluošto ir epoksido kompozito ilgų cilindrų, apkrautų išoriniu hidrauliniu slėgiu, klupimo procesas tiriamas deterministine ir tikimybine baigtinių elementų analize. Kritinis klupimo slėgis prognozuotas deterministine baigtinių elementų analize, Monte Karlo imitacija ir paviršiaus reakcijos metodu, ANSYS tikimybinio projektavimo sistema ir palygintas su eksperimentų rezultatais. Taip pat tirtas medžiagų savybių ir fizinių matmenų paklaidų poveikis kritiniam slėgiui. Tyrimo rezultatai rodo, kad deterministinė ir tikimybinė baigtinių elementų analizė prognozuoja panašius kritinius klupdymo slėgius, kurie yra truputį didesni už eksperimentinį. Klupimo tikimybė, prognozuota paviršiaus reakcijos metodu, atitinka eksperimentinį klupdymo slėgį. Kompozito sluoksnio storis, išilginis ir skersinis tamprumo moduliai ir vidinis spindulys žymų poveikį ilgų kompozicinių cilindrų darbui, tuo tarpu šlyties modulis, Puasono koeficientas, išorinių ir vidinių sluoksnių armavimo kampai ir neatremtas ilgis turi nedidelę įtaką.

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BUCKLING ANALYSIS OF COMPOSITE LONG CYLINDERS USING PROBABILISTIC FINITE ELEMENT METHOD

Summary

The buckling behaviours of filament-wound carbon fibre-epoxy composite long cylinders subjected to external hydrostatic pressure are researched by using deterministic and probabilistic finite element analyses. The critical buckling pressures predicted by deterministic finite element analysis and Monte Carlo simulation and response surface method of ANSYS probabilistic design system are compared with the experimental results. The effects of uncertainties of material properties and physical dimensions on the critical buckling pressure are also researched. The results show that the deterministic and probabilistic finite element analyses predict the similar critical buckling pressures, which are a little higher than the experimental results. The probability of buckling predicted by using response surface method is correct according to the experimental buckling pressure. The thickness of composite layers, transversal modulus, inside radius and longitudinal modulus have significant effects on the performance of composite long cylinders, whereas shear modulus, Poisson's ratio, winding angle of outer layers, winding angle of inner layers and unsupported length have small influences.

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